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Prime Conditions for Integers

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In this paper we suggest to break down natural numbers, P, in two coefficients, i and j, reported to their subscript N which is defined by the equation $P = 6N \pm 1$; i and j are defined by an other equation, $N = 6 * |i| * |j| \pm (i \pm j)$. N is a natural whole number but i and j are not necessarily natural whole numbers. They could be irrational. By using such unusual approach of number theory, we would propose a simple relation between two square numbers as a necessary condition for any prime number. We would like to suggest that such relationship could be looked as a corollary of the last Fermat's theorem.

Keywords: natural number, prime numbers, composite number, rational, irrational, Fermat's theorem

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INTRODUCTION

According to A. Maïsseu and B. Maïsseu [1], for any number $P \in F_P$, having the form $P = 6N \pm 1$, it is possible to find i and j such that $N = 6 * |i| * |j| \pm (i \pm j)$. This rule implicitly assumes that i and j are natural numbers:

$$i, j \in Z = \{-3, -2, -1, 1, 2, 3, 4, ...\}.$$

i and j are called the coefficients of subscript N_P, of a number P:

- When i and j are natural numbers, strictly rational, P is a composite number;
- When i and j are irrational, P is a prime number.

We have $S = i \pm j$, et $\Pi = |i| * |j|$.

According to the relation between the sum and the product of the roots of the equation of second degree

$$N = 6\,\Pi \pm S,$$

i and j are the roots of the second degree equation.

$$X^2 - SX + \Pi = 0.$$

With Δ called the determinant of such second degree equation

$$\Delta = S^2 - 4\Pi \geq 0$$

there comes the condition for P to be a prime number which is proposed below.

1. THE FUNDAMENTAL EQUATION

All prime numbers, $P > 3$, can be written in the form

$$P = 6N \pm 1$$

with N, called suscript of the number P, defined by the equation:

$$P = (\pm 1) \bmod 6.$$

However, not all numbers having this form are prime numbers. This family of numbers, F_P, hence comprises all prime numbers and composite numbers.

2. TWO SUB-SETS $\mathcal{F}_{\mathrm{V}}$ AND $\mathcal{F}_{\mathrm{VII}}$

The prime and composites numbers defined by the equation $P = 6N \pm 1$ can be broken down into two subsets.

1. $\mathcal{F}_{\mathrm{V}}$ for $P = 6N - 1$, i and j have different signs; their product is negative, $\Pi(i, j) < 0$.

2. $\mathcal{F}_{\mathrm{VII}}$ for $P = 6N + 1$, i and j have the same sign; their product is positive, $\Pi(i, j) > 0$.

Each of the two sub-sets can be broken down into six sub-families.

2.1. $\mathcal{F}_{\mathrm{V}}$: $P = 6N - 1$; i and j have different signs; $\Pi(i, j) = i * j < 0$

The six sub-families of $\mathcal{F}_{\mathrm{V}}$ are:

2.1.1. $P \in \mathcal{F}_{\mathrm{V}}$ [−1] with $P = (-1) + k * 36 \rightarrow P \equiv (-1) \bmod 36 \rightarrow N \equiv 0 \bmod 6$

P	N	I	J	Δ	$P = 9\Delta - \varkappa^2$	X = 6i − 1	Y = 6j + 1
−1	0	$\sqrt{0}$	$-\sqrt{0}$	0	$0 = 0 - 1^2$	$(\sqrt{0} * 6) - 1$	$(\sqrt{0} * 6) + 1$
35	6	1	−1	2^2	$35 = 6^2 - 1^2$	5	7
71	12	$\sqrt{2}$	$-\sqrt{2}$	8	$71 = 72 - 1^2$	$(\sqrt{2} * 6) - 1$	$(\sqrt{2} * 6) + 1$
107	18	$\sqrt{3}$	$-\sqrt{3}$	12	$107 = 108 - 1^2$	$(\sqrt{3} * 6) - 1$	$(\sqrt{3} * 6) + 1$

143	24	2	–2	16	$143 = 12^2 - 1^2$	11	13
179	30	$\sqrt{5}$	$-\sqrt{5}$	20	$179 = 180 - 1^2$	$(\sqrt{5} * 6) - 1$	$(\sqrt{5} * 6) + 1$
215	36	1	–7	8^2	$215 = 24^2 - 19^2$	5	43
251	42	$\sqrt{7}$	$-\sqrt{7}$	28	$251 = 252 - 1^2$	$(\sqrt{7} * 6) - 1$	$(\sqrt{7} * 6)+1$
287	48	7	–1	8^2	$287 = 24^2 - 17^2$	7	41
323	54	3	–3	6^2	$323 = 18^2 - 1^2$	17	19
359	60	$\sqrt{10}$	$-\sqrt{10}$	40	$359 = 360 - 1^2$	$(\sqrt{10} * 6) - 1$	$(\sqrt{10} * 6) + 1$
395	66	1	–13	14^2	$395 = 42^2 - 37^2$	5	79
431	72	$\sqrt{12}$	$-\sqrt{12}$	48	$431 = 432 - 1^2$	$(\sqrt{12} * 6) - 1$	$(\sqrt{12} * 6) + 1$

aso…

2.1.2. $P \in \mathscr{F}_V$ [5] with $P = 5 + k * 36 \rightarrow P \equiv$ $\equiv 5 \bmod 36 \rightarrow N \equiv 1 \bmod 6$

P	N	I	J	Δ	$P = 9\Delta - \varkappa^2$	$X = 6i - 1$	$Y = 6j + 1$
5	1	1	0	1	$5 = 3^2 - 2^2$	1	5
41	7	$(1 + \sqrt{5})/2$	$(1 - \sqrt{5})/2$	5	$41 = 45 - 2^2$	$(1 + \sqrt{5}) * 3 - 1$	$(1 - \sqrt{5}) * 3 + 1$
77	13	2	–1	9	$77 = 9^2 - 2^2$	7	11
113	19	$(1 + \sqrt{13})/2$	$(1 - \sqrt{13})/2$	13	$113 = 117 - 2^2$	$(1 + \sqrt{13}) * 3 - 1$	$(1 - \sqrt{13}) * 3 + 1$
149	25	$(1 + \sqrt{17})/2$	$(1 - \sqrt{17})/2$	17	$149 = 153 - 2^2$	$(1 + \sqrt{17}) * 3 - 1$	$(1 - \sqrt{17}) * 3 + 1$
185	31	1	–6	21	$185 = 21^2 - 16^2$	5	37
221	37	3	–2	25	$221 = 15^2 - 2^2$	13	17
257	43	$(1 + \sqrt{29})/2$	$(1 - \sqrt{29})/2$	29	$257 = 261 - 2^2$	$(1 + \sqrt{29}) * 3 - 1$	$(1 - \sqrt{29}) * 3 + 1$
293	49	$(1 + \sqrt{33})/2$	$(1 - \sqrt{33})/2$	33	$293 = 297 - 2^2$	$(1 + \sqrt{33}) * 3 - 1$	$(1 - \sqrt{33}) * 3 + 1$
329	55	8	–1	37	$329 = 27^2 - 20^2$	7	47
365	61	1	–12	41	$365 = 39^2 - 34^2$	5	73
401	67	$(1 + \sqrt{45})/2$	$(1 - \sqrt{45})/2$	45	$401 = 405 - 2^2$	$(1 + \sqrt{45}) * 3 - 1$	$(1 - \sqrt{45}) * 3 + 1$

aso…

2.1.3. $P \in \mathscr{F}_V$ [11] with $P = 11 + k * 36 \rightarrow P \equiv$ $\equiv 11 \bmod 36 \rightarrow N \equiv 2 \bmod 6$

P	N	i	J	Δ	$P = 9\Delta - \varkappa^2$	X	Y
11	2	2	0	2^2	$11 = 6^2 - 5^2$	1	11
47	8	$(1 + \sqrt{2})$	$(1 - \sqrt{2})$	8	$47 = 72 - 5^2$	$[1 + \sqrt{2}] * 6 - 1$	$[1 - \sqrt{2}] * 6 + 1$
83	14	$(1 + \sqrt{3})$	$(1 - \sqrt{3})$	12	$83 = 108 - 5^2$	$[1 + \sqrt{3}] * 6 - 1$	$[1 - \sqrt{3}] * 6 + 1$
119	20	3	–1	4^2	$119 = 12^2 - 5^2$	7	17
155	26	1	–5	6^2	$155 = 18^2 - 13^2$	5	31
191	32	$(1 + \sqrt{6})$	$(1 - \sqrt{6})$	24	$191 = 216 - 5^2$	$[1 + \sqrt{6}] * 6 - 1$	$[1 - \sqrt{6}] * 6 + 1$
227	38	$(1 + \sqrt{7})$	$(1 - \sqrt{7})$	28	$227 = 252 - 5^2$	$[1 + \sqrt{7}] * 6 - 1$	$[1 - \sqrt{7}] * 6 + 1$

263	44	$(1 + \sqrt{8})$	$(1 - \sqrt{8})$	32	$263 = 288 - 5^2$	$[1 + \sqrt{8}] * 6 - 1$	$[1 - \sqrt{8}] * 6 + 1$
299	50	4	–2	6^2	$299 = 18^2 - 5^2$	13	23
335	56	1	–11	12^2	$335 = 36^2 - 31^2$	5	67
371	62	–1	9	10^2	$371 = 30^2 - 23^2$	7	53
407	68	2	–6	8^2	$407 = 24^2 - 13^2$	11	37

aso…

2.1.4. $P \in \mathscr{F}_V$ [17] with $P = 17 + k * 36 \rightarrow P \equiv$ $\equiv 17 \bmod 36 \rightarrow N \equiv 3 \bmod 6$

P	N	i	j	Δ	$P = 9\Delta - \varkappa^2$	X	Y
17	3	3	0	3^2	$17 = 9^2 - 8^2$	6 * 8 – 1	0 * 0 + 1
53	9	$(3 + \sqrt{13})/2$	$(3 - \sqrt{13})/2$	$13 = 3^2 + 4 * 1$	$53 = 117 - 8^2$	$[3 + \sqrt{13}] * 3 - 1$	$[3 - \sqrt{13}] * 3 + 1$
89	15	$(3 + \sqrt{17})/2$	$(3 - \sqrt{17})/2$	$17 = 3^2 + 4 * 2$	$89 = 153 - 8^2$	$[3 + \sqrt{17}] * 3 - 1$	$[3 - \sqrt{17}] * 3 + 1$
125	21	1	–4	5^2	$125 = 15^2 - 10^2$	5	25
161	27	4	–1	5^2	$161 = 15^2 - 8^2$	23	7
197	33	$(3 + \sqrt{29})/2$	$(3 - \sqrt{29})/2$	29	$197 = 261 - 8^2$	$[3 + \sqrt{29}) * 3 - 1$	$[3 - \sqrt{29}] * 3 + 1$
233	39	$(3 + \sqrt{33})/2$	$(3 - \sqrt{33})/2$	33	$233 = 297 - 8^2$	$[3 + \sqrt{33}] * 3 - 1$	$[3 - \sqrt{33}] * 3 + 1$
269	45	$(3 + \sqrt{37})/2$	$(3 - \sqrt{37})/2$	37	$269 = 333 - 8^2$	$[3 + \sqrt{37}] * 3 - 1$	$[3 - \sqrt{37}] * 3 + 1$
305	51	1	–10	11^2	$305 = 33^2 - 28^2$	5	61
341	57	2	–5	7^2	$341 = 21^2 - 10^2$	11	31
377	63	5	–2	7^2	$377 = 21^2 - 8^2$	29	13
413	69	10	–1	11^2	$413 = 33^2 - 26^2$	59	7

etc…

2.1.5. $P \in \mathscr{F}_V$ [23] with $P = 23 + k * 36 \rightarrow P \equiv$ $\equiv 23 \bmod 36 \rightarrow N \equiv 4 \bmod 6$

P	N	i	j	Δ	$P = 9\Delta - \varkappa^2$	X	Y
23	4	4	0	4^2	$23 = 12^2 - 11^2$	23	1
59	10	$2 + \sqrt{5}$	$2 - \sqrt{5}$	20	$59 = 180 - 11^2$	$(2 + \sqrt{5}) * 6 - 1$	$(2 - \sqrt{5}) * 6 + 1$
95	16	1	–3	4^2	$95 = 12^2 - 7^2$	5	19
131	22	$2 + \sqrt{7}$	$2 - \sqrt{7}$	28	$131 = 252 - 11^2$	$(2 + \sqrt{7}) * 6 - 1$	$(2 - \sqrt{7}) * 6 + 1$
167	28	$2 + \sqrt{8}$	$2 - \sqrt{8}$	32	$167 = 288 - 11^2$	$(2 + \sqrt{8}) * 6 - 1$	$(2 - \sqrt{8}) * 6 + 1$
203	34	5	–1	6^2	$203 = 18^2 - 11^2$	29	7
239	40	$2 + \sqrt{10}$	$2 - \sqrt{10}$	40	$239 = 360 - 11^2$	$(2 + \sqrt{10}) * 6 - 1$	$(2 - \sqrt{10}) * 6 - 1$
275	46	2	–4	6^2	$275 = 18^2 - 7^2$	11	25
311	52	$2 + \sqrt{12}$	$2 - \sqrt{12}$	48	$311 = 432 - 11^2$	$(2 + \sqrt{12}) * 6 - 1$	$(2 - \sqrt{12}) * 6 + 1$
347	58	$2 + \sqrt{13}$	$2 - \sqrt{13}$	52	$347 = 468 - 11^2$	$(2 + \sqrt{13}) * 6 - 1$	$(2 - \sqrt{13}) * 6 + 1$

383	64	$2+\sqrt{14}$	$2-\sqrt{14}$	56	$383 = 504 - 11^2$	$(2+\sqrt{14}) * 6 - 1$	$(2-\sqrt{14}) * 6 + 1$
419	70	$2+\sqrt{15}$	$2-\sqrt{15}$	60	$419 = 540 - 11^2$	$(2+\sqrt{15}) * 6 - 1$	$(2-\sqrt{15}) * 6 + 1$

etc...

2.1.6. *P* ∈ $\mathscr{F}_{V}$ [29] with *P* = 29 + *k* * 36 → *P* ≡ ≡ 29 mod 36 → *N* ≡ 5 mod 6

P	N	i	j	Δ	$P = 9\Delta - \varkappa^2$	X	Y
29	5	5	0	5^2	$29 = 15^2 - 14^2$	29	1
65	11	1	−2	3^2	$65 = 9^2 - 4^2$	5	13
101	17	$(5+\sqrt{33})/2$	$(5-\sqrt{33})/2$	33	$101 = 297 - 14^2$	$(5+\sqrt{33}) * 3 - 1$	$(5-\sqrt{33}) * 3 + 1$
137	23	$(5+\sqrt{37})/2$	$(5-\sqrt{37})/2$	37	$137 = 333 - 14^2$	$(5+\sqrt{37}) * 3 - 1$	$(5-\sqrt{37}) * 3 + 1$
173	29	$(5+\sqrt{41})/2$	$(5-\sqrt{41})/2$	41	$173 = 369 - 14^2$	$(5+\sqrt{41}) * 3 - 1$	$(5-\sqrt{41}) * 3 + 1$
209	35	2	−3	5^2	$209 = 15^2 - 4^2$	11	19
245	41	6	−1	7^2	$245 = 21^2 - 14^2$	35	7
281	47	$(5+\sqrt{53})/2$	$(5-\sqrt{53})/2$	53	$281 = 477 - 14^2$	$(5+\sqrt{53}) * 3 - 1$	$(5-\sqrt{53}) * 3 + 1$
317	53	$(5+\sqrt{57})/2$	$(5-\sqrt{57})/2$	57	$317 = 513 - 14^2$	$(5+\sqrt{57}) * 3 - 1$	$(5-\sqrt{57}) * 3 + 1$
353	59	$(5+\sqrt{61})/2$	$(5-\sqrt{61})/2$	61	$353 = 549 - 14^2$	$(5+\sqrt{61}) * 3 - 1$	$(5-\sqrt{61}) * 3 + 1$
389	65	$(5+\sqrt{65})/2$	$(5-\sqrt{65})/2$	65	$389 = 585 - 14^2$	$(5+\sqrt{65}) * 3 - 1$	$(5-\sqrt{65}) * 3 + 1$
425	71	3	−4	7^2	$425 = 21^2 - 4^2$	17	25

2.2. $\mathscr{F}_{VII}$: $P = 6N + 1$; *i and j have same sign*; $\Pi(i, j) = i * j > 0$

The six sub-families of $\mathscr{F}_{VII}$ are:

2.2.1. *P* ∈ $\mathscr{F}_{VII}$ [1] with *P* = 1 + *k* * 36 → *P* ≡ ≡ 1 mod 36 → *N* ≡ 0 mod 6

P	N	i	J	Δ	$P = -9\Delta -+ \varkappa^2$	X	Y
1	0	0	0	0	$1 = 1^2 - 9 * 0^2$	1	1
37	6	6	0	6^2	$37 = 19^2 - 9 * 6^2$	37	1
73	12	$3+\sqrt{8}$	$3-\sqrt{8}$	32	$73 = 19^2 - 8 * 36$	$(3+\sqrt{8}) * 6 + 1$	$\lvert(3-\sqrt{8})\rvert * 6 + 1$
109	18	$3+\sqrt{7}$	$3-\sqrt{7}$	28	$109 = 19^2 - 7 * 36$	$(3+\sqrt{7}) * 6 + 1$	$\lvert(3-\sqrt{7})\rvert * 6 + 1$
145	24	−5	−1	4^2	$145 = 17^2 - 12^2$	29	5
181	30	$3+\sqrt{5}$	$3-\sqrt{5}$	20	$181 = 19^2 - 5 * 36$	$(3+\sqrt{5}) * 6 + 1$	$\lvert(3-\sqrt{5})\rvert * 6 + 1$
217	36	5	1	4^2	$217 = 19^2 - 12^2$	7	31
253	42	−4	−2	2^2	$253 = 17^2 - 6^2$	23	11
289	48	−3	−3	0^2	$289 = 17^2 - 0^2$	17	17
325	54	4	2	2^2	$325 = 19^2 - 6^2$	25	13
	54	−11	−1	10^2	$325 = 35^2 - 30^2$	*65*	*5*
361	60	3	3	0^2	$361 = 19^2 - 0^2$	19	19
397	66	$6+\sqrt{27}$	$6-\sqrt{27}$	108	$397 = 37^2 - 27 * 36$	$(6+\sqrt{27}) * 6 + 1$	$\lvert(6-\sqrt{27})\rvert * 6 + 1$
433	72	$6+\sqrt{26}$	$6-\sqrt{26}$	104	$433 = 37^2 - 26 * 36$	$(6+\sqrt{26}) * 6 + 1$	$\lvert(6+\sqrt{26})\rvert * 6 + 1$

aso...

2.2.2. *P* ∈ $\mathscr{F}_{VII}$ [7] with *P* = (7) + *k* * 36 → *P* ≡ ≡ 7 mod 36 → *N* ≡ 1 mod 6

P	N	i	J	Δ	$P = \varkappa^2 - 9\Delta$	X	Y
7	1	1	0	1	$7 = 4^2 - 9 * 1$	7	1
43	7	$(-5+\sqrt{17})/2$	$(-5-\sqrt{17})/2$	17	$43 = 14^2 - 9 * 17$	$(\lvert-5+\sqrt{17}\rvert) * 3-1$	$(\lvert-5-\sqrt{17}\rvert) * 3-1$
79	13	$(-5+\sqrt{13})/2$	$(-5-\sqrt{13})/2$	13	$79 = 14^2 - 9 * 13$	$(\lvert-5+\sqrt{13}\rvert) * 3 - 1$	$(\lvert-5-\sqrt{13}\rvert) * 3 - 1$
115	19	−1	−4	3^2	$115 = 14^2 - 9^2$	5	23
151	25	$(-5+\sqrt{5})/2$	$(-5-\sqrt{5})/2$	5	$151 = 14^2 - 9 * 5$	$(\lvert-5+\sqrt{5}\rvert) * 3 - 1$	$(\lvert-5-\sqrt{5}\rvert) * 3 - 1$
187	31	−2	−3	1	$187 = 14^2 - 3^2$	11	17
223	37	$(-11+\sqrt{89})/2$	$(-11-\sqrt{89})/2$	89	$223 = 32^2 - 9 * 89$	$(\lvert-11+\sqrt{89}\rvert) * 3 - 1$	$(\lvert-11-\sqrt{89}\rvert) * 3 - 1$
259	43	1	6	5^2	$259 = 22^2 - 15^2$	7	37
295	49	−1	−10	9^2	$295 = 32^2 - 27^2$	5	59
331	55	$(-11+\sqrt{77})/2$	$(-11-\sqrt{77})/2$	77	$331 = 32^2 - 9 * 77$	$(\lvert-11+\sqrt{77}\rvert) * 3 - 1$	$(\lvert-11-\sqrt{77}\rvert) * 3 - 1$
367	61	$(-11+\sqrt{73})/2$	$(-11-\sqrt{73})/2$	7	$367 = 32^2 - 9 * 73$	$(\lvert-11+\sqrt{73}\rvert) * 3 - 1$	$(\lvert-11-\sqrt{73}\rvert) * 3 - 1$
403	67	2	5	3^2	$403 = 22^2 - 9^2$	13	31

aso...

2.2.3. *P* ∈ $\mathscr{F}_{VII}$ [13] with *P* = (13) + *k* * 36 → *P* ≡ ≡ 13 mod 36 → *N* ≡ 2 mod 6

P	N	i	J	Δ	$P = \varkappa^2 - 9\Delta$	X	Y
13	2	2	0	2^2	$13 = 7^2 - 6^2$	13	1
49	8	1	1	0	$49 = 7^2 - 3^2$	7	7
85	14	−3	−1	2^2	$85 = 11^2 - 6^2$	17	5
121	20	−2	−2	4^2	$121 = 11^2 - 3^2$	11	11
157	26	$(-8+\sqrt{52})/2$	$(-8-\sqrt{52})/2$	52	$157 = 25^2 - 9 * 52$	$\lvert(-8+\sqrt{52})\rvert * 3 + 1$	$\lvert(-8-\sqrt{52})\rvert * 3 + 1$
193	32	$(-8+\sqrt{48})/2$	$(-8-\sqrt{48})/2$	48	$193 = 25^2 - 9 * 48$	$\lvert(-8+\sqrt{48})\rvert * 3 + 1$	$\lvert(-8-\sqrt{48})\rvert * 3 + 1$
229	38	$(-8+\sqrt{44})/2$	$(-8-\sqrt{44})/2$	44	$229 = 25^2 - 9 * 44$	$\lvert(-8+\sqrt{44})\rvert * 3 + 1$	$\lvert(-8-\sqrt{44})\rvert * 3 + 1$
265	44	−9	−1	8^2	$265 = 29^2 - 24^2$	53	5
301	50	7	1	6^2	$301 = 25^2 - 18^2$	43	7
337	56	$(-8+\sqrt{32})/2$	$(-8-\sqrt{32})/2$	32	$337 = 25^2 - 9 * 32$	$\lvert(-8+\sqrt{32})\rvert * 3 + 1$	$\lvert(-8-\sqrt{32})\rvert * 3 + 1$

373	62	$(-8+\sqrt{28})/2$	$(-8-\sqrt{28})/2$	28	$373 = 25^2 - 9 * 28$	$\lvert(-8+\sqrt{28})\rvert * 3 + 1$	$\lvert(-8-\sqrt{28})\rvert * 3 + 1$
409	68	$(-8+\sqrt{24})/2$	$(-8-\sqrt{24})/2$	24	$409 = 25^2 - 9 * 24$	$\lvert(-8+\sqrt{24})\rvert * 3 + 1$	$\lvert(-8-\sqrt{24})\rvert * 3 + 1$

aso ...

2.2.4. $P \in \mathcal{F}_{VII}$ [19] with $P = (19) + k * 36 \rightarrow P \equiv$ $\equiv 19 \bmod 36 \rightarrow N \equiv 3 \bmod 6$

P	N	i	J	Δ	$P = \varkappa^2 - 9\Delta$	X	Y
19	3	3	0	3^2	$19 = 10^2 - 9^2$	19	1
55	9	−1	−2	1	$55 = 8^2 - 3^2$	5	11
91	15	1	2	1	$91 = 10^2 - 3^2$	7	13
127	21	$(9+\sqrt{73})/2$	$(9-\sqrt{73})/2$	73	$127 = 28^2 - 9 * 73$	$(9+\sqrt{73}) * 3 + 1$	$(9-\sqrt{73}) * 3 + 1$
163	27	$(9+\sqrt{69})/2$	$(9-\sqrt{69})/2$	69	$163 = 28^2 - 9 * 69$	$(9+\sqrt{69}) * 3 + 1$	$(9-\sqrt{69}) * 3 + 1$
199	33	$(9+\sqrt{65})/2$	$(9-\sqrt{65})/2$	65	$199 = 28^2 - 9 * 65$	$(9+\sqrt{65}) * 3 + 1$	$(9-\sqrt{65}) * 3 + 1$
235	39	−1	−8	7^2	$235 = 26^2 - 21^2$	5	47
271	45	$(9+\sqrt{57})/2$	$(9-\sqrt{57})/2$	57	$271 = 28^2 - 9 * 57$	$(9+\sqrt{57}) * 3 + 1$	$(9-\sqrt{57}) * 3 + 1$
307	51	$(9+\sqrt{53})/2$	$(9-\sqrt{53})/2$	55	$307 = 28^2 - 9 * 53$	$(9+\sqrt{53}) * 3 + 1$	$(9-\sqrt{53}) * 3 + 1$
343	57	1	8	7^2	$343 = 28^2 - 21^2$	7	49
379	63	$(9+\sqrt{45})/2$	$(9-\sqrt{45})/2$	45	$379 = 28^2 - 9 * 45$	$(9+\sqrt{45}) * 3 + 1$	$(9-\sqrt{45}) * 3 + 1$
415	69	−1	−14	13^2	$415 = 44^2 - 39^2$	5	83

aso...

2.2.5. $P \in \mathcal{F}_{VII}$ [25] with $P = (25) + k * 36 \rightarrow P \equiv$ $\equiv 25 \bmod 36 \rightarrow N \equiv 4 \bmod 6$

P	N	i	J	Δ	$P = \varkappa^2 - 9\Delta$	X	Y
25	4	0	4	4^2	$25 = 13^2 - 12^2$	1	25
61	10	$(-4+\sqrt{13})$	$(-4-\sqrt{13})$	52	$61 = 23^2 - 9 * 52$	$(-4+\sqrt{13}) * 6 - 1$	$(-4-\sqrt{13}) * 6 - 1$
97	16	$(-4+\sqrt{12})$	$(-4-\sqrt{12})$	48	$97 = 23^2 - 9 * 48$	$(-4+\sqrt{12}) * 6 - 1$	$(-4-\sqrt{12}) * 6 - 1$
133	22	1	3	2^2	$133 = 13^2 - 6^2$	7	19
169	28	2	2	0	$169 = 13^2 - 9 * 0$	13	13
205	34	−1	−7	6^2	$205 = 23^2 - 18^2$	5	41
241	40	$(-4+\sqrt{8})$	$(-4-\sqrt{8})$	32	$241 = 23^2 - 9 * 32$	$(-4+\sqrt{8}) * 6 - 1$	$(-4-\sqrt{8}) * 6 - 1$
277	46	$(-4+\sqrt{7})$	$(-4-\sqrt{7})$	28	$277 = 23^2 - 9 * 28$	$(-4+\sqrt{7}) * 6 - 1$	$(-4-\sqrt{7}) * 6 - 1$
313	52	$(-4+\sqrt{6})$	$(-4-\sqrt{6})$	24	$313 = 23^2 - 9 * 24$	$(-4+\sqrt{6}) * 6 - 1$	$(-4-\sqrt{6}) * 6 - 1$
349	58	$(-4+\sqrt{5})$	$(-4-\sqrt{5})$	20	$349 = 23^2 - 9 * 20$	$(-4+\sqrt{5}) * 6 - 1$	$(-4-\sqrt{5}) * 6 - 1$
385	64	−1	−13	12^2	$385 = 41^2 - 36^2$	5	77
		1	*9*	8^2	$385 = 31^2 - 24^2$	*7*	*55*
		−2	*−6*	4^2	$385 = 23^2 - 12^2$	*11*	*35*
421	70	$(-4+\sqrt{3})$	$(-4-\sqrt{3})$	12	$421 = 23^2 - 9 * 12$	$(-4+\sqrt{3}) * 6 - 1$	$(-4-\sqrt{3}) * 6 - 1$

aso...

2.2.6. $P \in \mathcal{F}_{VII}$ [31] with $P = (31) + k * 36 \rightarrow P \equiv$ $\equiv 31 \bmod 36 \rightarrow N \equiv 5 \bmod 6$

P	N	i	J	Δ	$P = \varkappa^2 - 9\Delta$	X	Y
31	5	5	0	5^2	$31 = 16 - 9 * 25$	31	1
67	11	$(-7+\sqrt{37})/2$	$(-7-\sqrt{37})/2$	37	$67 = 20^2 - 9 * 37$	$\lvert(-7+\sqrt{37})\rvert * 3 - 1$	$\lvert(-7-\sqrt{37})\rvert * 3 - 1$
103	17	$(-7+\sqrt{33})/2$	$(-7-\sqrt{33})/2$	33	$103 = 20^2 - 9 * 33$	$\lvert(-7+\sqrt{33})\rvert * 3 - 1$	$\lvert(-7-\sqrt{33})\rvert * 3 - 1$
139	23	$(-7+\sqrt{29})/2$	$(-7-\sqrt{29})/2$	29	$133 = 20^2 - 9 * 29$	$\lvert(-7+\sqrt{29})\rvert * 3 - 1$	$\lvert(-7-\sqrt{29})\rvert * 3 - 1$
175	29	−1	−6	5^2	$175 = 20^2 - 9 * 25$	5	35
	29	*1*	*4*	3^2	$175 = 16^2 - 9 * 9$	*7*	*25*
211	35	$(-7+\sqrt{21})/2$	$(-7-\sqrt{21})/2$	21	$211 = 20^2 - 9 * 21$	$\lvert(-7+\sqrt{21})\rvert * 3 - 1$	$\lvert(-7-\sqrt{21})\rvert * 3 - 1$
247	41	2	3	1^2	$247 = 16^2 - 9 * 1$	13	19
283	47	$(-7+\sqrt{13})/2$	$(-7-\sqrt{13})/2$	13	$283 = 20^2 - 9 * 13$	$\lvert(-7+\sqrt{13})\rvert * 3 - 1$	$\lvert(-7-\sqrt{13})\rvert * 3 - 1$
319	53	−2	−5	3^2	$319 = 20^2 - 9 * 9$	11	29
355	59	−1	−12	11^2	$355 = 38^2 - 9 * 121$	5	71
391	65	−3	−4	1^2	$391 = 20^2 - 1 * 9$	17	23
427	71	1	10	9^2	$427 = 34^2 - 9 * 81$	7	61

aso ...

2.3. Such relations are insufficient for qualifying prime number

It is easy to check that these relations are insufficient to check the quality of any number as prime or composite.

3. SOME RELATIONS: DIFFERENCE OF TWO SQUARE NUMBERS

Regarding Stirling:

- $Y^2 - X^2 = (Y - X) + 2\,[C_{(2,Y)} - C_{(2,X)}]$;
- $(n + 1)^2 - n^2 = 2n + 1$;
- The difference between two square consecutive numbers is an odd number; reciprocally, any odd number is the difference between two square consecutive numbers.

$$(n + 1)^2 - n^2 = 2n + 1$$

Hence the well-known rule:

"Any number is equal to the difference between two square numbers if it is the sum of consecutive odd numbers."

For example:

$40 = 7^2 - 3^2 = 13 + 11 + 9 + 7 = [(13 + 7)/2] * 4 = 40.$

4. PRIME CONDITIONS FOR P-

4.1. Determination of indice u

$$u = (i + j + 2)/6,$$
$$i + j = 6u - 2.$$

So what:

$$N = 6ij + i + j = 6ij + 6u - 2.$$

4.2. The co-first indice M'_P

4.2.1.

Here is Δ, determinant of the second degree equation describing the relation between the sum and the product of the coefficients *i* and *j* of N_P, subscript of P.

So what

$$\Delta = S^2 - 4\Pi.$$

We have also

$$N = 6\Pi \pm S.$$

P is a composite number, if Δ is square number. If not, P is a prime number; the roots *i* and *j* of the second degree equation, which are the coefficients of N_P, are irrational.

Looking for the prime quality of *P*, is looking for Δ as a square number.

Example:

$$P = 2783, \quad N = 464; \quad (i, j) = [4, -20].$$

We are going to use the well known incremental process; the dividend, N_P (also the subscript o *P*) et and the divider (6) stay unchanged.

Example:

- $N = 464 = 6 * 77 + 2$ bar $\Delta = 2^2 - 4(-77) = 312$ that's not a square number;
- $N = 464 = 6 * 78 - 4$ bar $\Delta = 4^2 - 4(-78) = 328$ that's not a square number;
- $N = 464 = 6 * 79 - 10$ bar $\Delta = 10^2 - 4(-79) = 416$ that's not a square number;
- $N = 464 = 6 * 80 - 16$ bar $\Delta = 16^2 - 4(-80) = 576 = 312 + 264 = 24^2$.

It is easy to check $264 = 16C_{(1,u)} + 72C_{(2,u)} =$ Щ with $u = 3$ and $312 = (-8) + 68C_{(1,i)} + 8C_{(2,i)} = M'_P$ with $i = 4$ and $j = (-20)$.

That means: $\Delta = M'_P +$ Щ $= 312 + 264 = 576 = 24^2$.

4.2.2. Relationships for co-first indice M'_P

4.2.2.1. For $P \in \mathcal{F}_V$: $P = 6N - 1$; *i* and *j* have opposite signs, $\Pi < 0$

- $P \in \mathcal{F}_V[-1]$ with $P = (-1) + k * 36$

For $i > 0$, $j < 0$ and $j > i$, that's to say $S < 0$; here comes for M'_P:

$(-4) + 28C_{(1,i)} + 8C_{(2,i)}$	$(-8) + 52C_{(1,i)} + 8C_{(2,i)}$	$(-12) + 76C_{(1,i)} + 8C_{(2,i)}$	$(-16) + 100C_{(1,i)} + 8C_{(2,i)}$	$(-20) + 124C_{(1,i)} + 8C_{(2,i)}$	Etc...

For $i < 0$, $j > 0$ and $j > i$, that's to say $\boldsymbol{S > 0}$; here comes for M'_P:

$0 + 4C_{(1,i)} + 8C_{(2,i)}$	$4 + 28C_{(1,i)} + 8C_{(2,i)}$	$8 + 52C_{(1,i)} + 8C_{(2,i)}$	$12 + 76C_{(1,i)} + 8C_{(2,i)}$	$16 + 100C_{(1,i)} + 8C_{(2,i)}$	Etc...

- $P \in \mathcal{F}_V[5]$ with $P = 5 + k * 36$

For $i > 0$, $j < 0$ and $j > i$, that's to say $S < 0$; here comes for M'_P:

$(-3) + 24C_{(1,i)} + 8C_{(2,i)}$	$(-7) + 48C_{(1,i)} + 8C_{(2,i)}$	$(-11) + 72C_{(1,i)} + 8C_{(2,i)}$	$(-15) + 96C_{(1,i)} + 8C_{(2,i)}$	$(-19) + 120C_{(1,i)} + 8C_{(2,i)}$	Etc...

For $i < 0$, $j > 0$ and $j > i$, that's to say $\boldsymbol{S > 0}$; here comes for M'_P:

$1 + 8C_{(1,i)} + 8C_{(2,i)}$	$5 + 32C_{(1,i)} + 8C_{(2,i)}$	$9 + 56C_{(1,i)} + 8C_{(2,i)}$	$13 + 80C_{(1,i)} + 8C_{(2,i)}$	$17 + 104C_{(1,i)} + 8C_{(2,i)}$	Etc...

- $P \in \mathcal{F}_V[11]$ avec $P = 11 + k * 36$

For $i > 0$, $j < 0$ and $j > i$, that's to say $S < 0$; here comes for M'_P:

$0 + 20C_{(1,i)} + 8C_{(2,i)}$	$(-4) + 44C_{(1,i)} + 8C_{(2,i)}$	$(-8) + 68C_{(1,i)} + 8C_{(2,i)}$	$(-12) + 92C_{(1,i)} + 8C_{(2,i)}$	$(-16) + 116C_{(1,i)} + 8C_{(2,i)}$	Etc...

For $i < 0$, $j > 0$ and $j > i$, that's to say $S > 0$; here comes for M'_P:

$4 + 12C_{(1,i)} + 8C_{(2,i)}$	$8 + 36C_{(1,i)} + 8C_{(2,i)}$	$12 + 60C_{(1,i)} + 8C_{(2,i)}$	$16 + 84C_{(1,i)} + 8C_{(2,i)}$	$20 + 108C_{(1,i)} + 8C_{(2,i)}$	Etc...

- $P \in \mathcal{F}_V[17]$ avec $P = 17 + k * 36$

For $i > 0$, $j < 0$ and $j > i$, that's to say $S < 0$; here comes for M'_P:

$5 + 16C_{(1,i)} + 8C_{(2,i)}$	$1 + 40C_{(1,i)} + 8C_{(2,i)}$	$(-3) + 64C_{(1,i)} + 8C_{(2,i)}$	$(-7) + 88C_{(1,i)} + 8C_{(2,i)}$	$(-11) + 112C_{(1,i)} + 8C_{(2,i)}$	Etc...

For $i < 0$, $j > 0$ and $j > i$, that's to say $S > 0$; here comes for M'_P:

$9 + 16C_{(1,i)} + 8C_{(2,i)}$	$13 + 40C_{(1,i)} + 8C_{(2,i)}$	$17 + 64C_{(1,i)} + 8C_{(2,i)}$	$21 + 88C_{(1,i)} + 8C_{(2,i)}$	$25 + 112C_{(1,i)} + 8C_{(2,i)}$	Etc...

• $P \in \mathcal{F}_V$ [23] avec $P = 23 + k * 36$

For $i > 0$, $j < 0$ and $j > i$, that's to say $\boldsymbol{S < 0}$; here comes for M'_P:

$4 + 12C_{(1,i)} + 8C_{(2,i)}$	$0 + 36C_{(1,i)} + 8C_{(2,i)}$	$(-4) + 60C_{(1,i)} + 8C_{(2,i)}$	$(-8) + 84C_{(1,i)} + 8C_{(2,i)}$	$(-12) + 108C_{(1,i)} + 8C_{(2,i)}$	Etc...

For $i < 0$, $j > 0$ and $j > i$, that's to say $\boldsymbol{S > 0}$; here comes for M'_P:

$8 + 20C_{(1,i)} + 8C_{(2,i)}$	$12 + 44C_{(1,i)} + 8C_{(2,i)}$	$16 + 68C_{(1,i)} + 8C_{(2,i)}$	$20 + 92C_{(1,i)} + 8C_{(2,i)}$	$24 + 116C_{(1,i)} + 8C_{(2,i)}$	Etc...

• $P \in \mathcal{F}_V$ [29] avec $P = 29 + k * 36$

For $i > 0$, $j < 0$ and $j > i$, that's to say $\boldsymbol{S < 0}$; here comes for M'_P:

$1 + 8C_{(1,i)} + 8C_{(2,i)}$	$(-3) + 32C_{(1,i)} + 8C_{(2,i)}$	$(-7) + 56C_{(1,i)} + 8C_{(2,i)}$	$(-11) + 80C_{(1,i)} + 8C_{(2,i)}$	$(-15) + 104C_{(1,i)} + 8C_{(2,i)}$	Etc...

For $i < 0$, $j > 0$ and $j > i$, that's to say $\boldsymbol{S > 0}$; here comes for M'_P:

$5 + 24C_{(1,i)} + 8C_{(2,i)}$	$9 + 48C_{(1,i)} + 8C_{(2,i)}$	$13 + 72C_{(1,i)} + 8C_{(2,i)}$	$17 + 96C_{(1,i)} + 8C_{(2,i)}$	$21 + 120C_{(1,i)} + 8C_{(2,i)}$	Etc...

4.2.2.2. For P ∈ $\mathcal{F}_{VII}$: $P = 6N + 1$ 1; i and j have same sign, $\Pi > 0$

• P ∈ $\mathcal{F}_{VII}$ [1] avec $P = 1 + k * 36$

For $i > 0$, $j > 0$; that's to say $S > 0$, here comes for M'_P:

$(-4)-20C_{(1,i)} + 8C_{(2,i)}$	$(-8)-44C_{(1,i)} + 8C_{(2,i)}$	$(-12)-68C_{(1,i)} + 8C_{(2,i)}$	$(-16)-92C_{(1,i)} + 8C_{(2,i)}$	$(-20)-116C_{(1,i)} + 8C_{(2,i)}$	Etc...

For $i < 0$, $j < 0$; that's to say $\boldsymbol{S < 0}$, here comes for M'_P:

$4-20C_{(1,i)} + 8C_{(2,i)}$	$8-44C_{(1,i)} + 8C_{(2,i)}$	$12-68C_{(1,\mathrm{i})} + 8C_{(2,\mathrm{i})}$	$16-92C_{(1,i)} + 8C_{(2,i)}$	$20-116C_{(1,i)} + 8C_{(2,i)}$	Etc...

• $P \in \mathcal{F}_{VII}$ [7] avec $P = 7 + \mathrm{k}*36$

For $i > 0$, $j > 0$; that's to say $S > 0$, here comes for M'_P:

$(-3)-24C_{(1,i)} + 8C_{(2,i)}$	$(-7)-48C_{(1,i)} + 8C_{(2,i)}$	$(-11)-72C_{(1,i)} + 8C_{(2,i)}$	$(-15)-96C_{(1,i)} + 8C_{(2,i)}$	$(-19)-120C_{(1,i)} + 8C_{(2,i)}$	Etc...

For $i < 0$, $j < 0$; that's to say $S < 0$, here comes for M'_P:

$5-16C_{(1,i)} + 8C_{(2,i)}$	$9-40C_{(1,i)} + 8C_{(2,i)}$	$13-64C_{(1,i)} + 8C_{(2,i)}$	$17-88C_{(1,i)} + 8C_{(2,i)}$	$21-112C_{(1,i)} + 8C_{(2,i)}$	Etc...

• $P \in \mathcal{F}_{VII}$ [13] avec $P = 13 + k * 36$

For $i > 0$, $j > 0$; that's to say $S > 0$, here comes for M'_P:

$0-28C_{(1,i)} + 8C_{(2,i)}$	$(-4)-52C_{(1,i)} + 8C_{(2,i)}$	$(-8)-76C_{(1,i)} + 8C_{(2,i)}$	$(-12)-100C_{(1,i)} + 8C_{(2,i)}$	$(-16)-124C_{(1,i)} + 8C_{(2,i)}$	Etc...

For $i < 0$, $j < 0$; that's to say $S < 0$, here comes for M'_P:

$8-12C_{(1,i)} + 8C_{(2,i)}$	$12-36C_{(1,i)} + 8C_{(2,i)}$	$16-60C_{(1,i)} + 8C_{(2,i)}$	$20-84C_{(1,i)} + 8C_{(2,i)}$	$24-108C_{(1,i)} + 8C_{(2,i)}$	Etc...

• $P \in \mathcal{F}_{VII}$ [19] avec $P = 19 + k * 36$

For $i > 0$, $j > 0$; that's to say $S > 0$, here comes for M'_P:

$5-32C_{(1,i)} + 8C_{(2,i)}$	$1-56C_{(1,i)} + 8C_{(2,i)}$	$(-3)-80C_{(1,i)} + 8C_{(2,i)}$	$(-7)-104C_{(1,i)} + 8C_{(2,i)}$	$(-11)-128C_{(1,i)} + 8C_{(2,i)}$	Etc...

For $i < 0, j < 0$; that's to say $S < 0$, here comes for M'_P:

$13-8C_{(1,i)} + 8C_{(2,i)}$	$17-32C_{(1,i)} + 8C_{(2,i)}$	$21-56C_{(1,i)} + 8C_{(2,i)}$	$25-80C_{(1,i)} + 8C_{(2,i)}$	$29-104C_{(1,i)} + 8C_{(2,i)}$	Etc...

- $P \in \mathcal{F}_{VII}$ [25] avec $P = 25 + k * 36$

For $i > 0, j > 0$; that's to say $S > 0$, here comes for M'_P:

$12-36C_{(1,i)} + 8C_{(2,i)}$	$8-60C_{(1,i)} + 8C_{(2,i)}$	$4-84C_{(1,i)} + 8C_{(2,i)}$	$0-108C_{(1,i)} + 8C_{(2,i)}$	$(-4)-132C_{(1,i)} + 8C_{(2,i)}$	Etc...

For $i < 0, j < 0$; that's to say $S < 0$, here comes for M'_P:

$24-28C_{(1,i)} + 8C_{(2,i)}$	$28-52C_{(1,i)} + 8C_{(2,i)}$	$32-76C_{(1,i)} + 8C_{(2,i)}$	$36-100C_{(1,i)} + 8C_{(2,i)}$	$40-124C_{(1,i)} + 8C_{(2,i)}$	Etc...

- $P \in \mathcal{F}_{VII}$ [31] avec $P = 31 + k * 36$

For $i > 0, j > 0$; that's to say $S > 0$, here comes for M'_P:

$21-40C_{(1,i)} + 8C_{(2,i)}$	$17-64C_{(1,i)} + 8C_{(2,i)}$	$13-88C_{(1,i)} + 8C_{(2,i)}$	$9-112C_{(1,i)} + 8C_{(2,i)}$	$5-136C_{(1,i)} + 8C_{(2,i)}$	Etc...

For $i < 0, j < 0$; that's to say $S < 0$, here comes for M'_P:

$29-0C_{(1,i)} + 8C_{(2,i)}$	$33-24C_{(1,i)} + 8C_{(2,i)}$	$37-48C_{(1,i)} + 8C_{(2,i)}$	$41-72C_{(1,i)} + 8C_{(2,i)}$	$45-96C_{(1,i)} + 8C_{(2,i)}$	Etc...

4.2.3. The co-second indice M''_P

M'_P, co-first indice of P, is composed with two elements, one of them is the sum of binomial coefficients.

We will call ψ_M the fisrt part and C_M the second part.

It comes:

$$M'_P = \psi_M + C_M$$

example for $\mathcal{F}_V$ [11], with $S<0$:

- for $u = 1$, it comes: $M'_P = 0 + 20C_{(1,i)} + 8C_{(2,i)}$, so $\psi = 0$;
- for $u = 2$ it comes: $M'_P = (-4) + 44C_{(1,i)} + 8C_{(2,i)}$, so $\psi = -4$;
- for $u = 3$, it comes: $M'_P = (-8) + 68C_{(1,i)} + 8C_{(2,i)}$, so $\psi = -8$;
- for $u = 4$, it comes: $M'_P = (-12) + 92C_{(1,i)} + 8C_{(2,i)}$, so $\psi = -12$;
- for $u = 5$, it comes: $M'_P = (-16) + 116C_{(1,i)} + 8C_{(2,i)}$, so $\psi = -16$.

		$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 5$
$u = 1$	$M'_P = 0 + 20C_{(1,i)} + 8C_{(2,i)}$	20	48	84	128	180
$u = 2$	$M'_P = (-4) + 44C_{(1,i)} + 8C_{(2,i)}$	40	92	152	220	296
$u = 3$	$M'_P = (-8) + 68C_{(1,i)} + 8C_{(2,i)}$	60	136	220	**312**	412
$u = 4$	$M'_P = (-12) + 92C_{(1,i)} + 8C_{(2,i)}$	80	180	288	404	528
$u = 5$	$M'_P = (-16) + 116C_{(1,i)} + 8C_{(2,i)}$	100	224	356	496	644
+						
		$u = 1$	$u = 2$	$u = 3$	$u = 4$	$u = 5$
	Щ $= 16\ C_{(1,\ u)} + 72\ C_{(2,\ u)}$	16	104	**264**	496	800
$= \Delta$						

Example:

for $P = 2783 \in \mathcal{F}_V$ [11] with $N_{2783} = 464$; $M'_{2783} = 312$, here is

$$\Delta = 264 + 312 = 24^2.$$

M_P could also be described as a function of indice u.

For example with $\mathcal{F}_V$ [11], it comes

$$M'_P = (-4)(u-1) + 4(6u-1)C_{(1,i)} + 8C_{(2,i)}.$$

We will call M''_P, co-second indice:

$$M''_P = M'_P - \psi_M.$$

By first, here comes the table of coefficients of ψ_M:

	$u = 1$	$u = 2$	$u = 3$	$u = 4$	$u = 5$	Etc ..
$P \in \mathcal{F}_V$ [–1] $S < 0$:	(–4)	(–8)	(–12)	(–16)	(–20)	Etc ..
$P \in \mathcal{F}_V$ [–1] $S > 0$	0	4	8	12	16	Etc...
$P \in \mathcal{F}_V$ [5] $S < 0$:	(–3)	(–7)	(–11)	(–15)	(–19)	Etc...
$P \in \mathcal{F}_V$ [5] $S > 0$	1	5	9	13	17	Etc...
$P \in \mathcal{F}_V$ [11] $S < 0$:	0	(–4)	(–8)	(–12)	(–16)	Etc...
$P \in \mathcal{F}_V$ [11] $S > 0$	4	8	12	16	20	Etc...
$P \in \mathcal{F}_V$ [17] $S < 0$	5	1	(–3)	(–7)	(–11)	Etc...
$P \in \mathcal{F}_V$ [17] $S > 0$	9	13	17	21	25	Etc...
$P \in \mathcal{F}_V$ [23] $S < 0$	4	0	(–4)	(–8)	(–12)	Etc...
$P \in \mathcal{F}_V$ [23] $S > 0$	8	12	16	20	24	Etc...
$P \in \mathcal{F}_V$ [29] $S < 0$	1	(–3)	(–7)	(–11)	(–15)	Etc...
$P \in \mathcal{F}_V$ [29] $S > 0$	5	9	13	17	21	Etc...
$P \in \mathcal{F}_{VII}$ [[1]] $S > 0$	(–4)	(–8)	(–12)	(–16)	(–20)	Etc...

$P \in \mathscr{F}_{\text{VII}}$ [1] $S < 0$	4	8	12	16	20	Etc...
$P \in \mathscr{F}_{\text{VII}}$ [7] $S > 0$	(–3)	(–7)	(–11)	(–15)	(–19)	Etc...
$P \in \mathscr{F}_{\text{VII}}$ [7] $S < 0$	5	9	13	17	21	Etc...
$P \in \mathscr{F}_{\text{VII}}$ [13] $S > 0$	0	(–4)	(–8)	(–12)	(–16)	Etc...
$P \in \mathscr{F}_{\text{VII}}$ [13] $S < 0$	8	12	16	20	24	Etc...
$P \in \mathscr{F}_{\text{VII}}$ [19] $S > 0$	5	1	(–3)	(–7)	(–11)	Etc...
$P \in \mathscr{F}_{\text{VII}}$ [19] $S < 0$	13	17	21	25	29	Etc...
$P \in \mathscr{F}_{\text{VII}}$ [25] $S > 0$	12	8	4	0	(–4)	Etc...
$P \in \mathscr{F}_{\text{VII}}$ [25] $S < 0$	24	28	32	36	40	Etc...
$P \in \mathscr{F}_{\text{VII}}$ [31] $S > 0$	21	17	13	9	5	Etc...
$P \in \mathscr{F}_{\text{VII}}$ [31] $S < 0$	29	33	37	41	45	Etc...

4.3. Calculation of $Щ_u$

(let remind about $Щ_u$: its definition via $\Delta = M'_P + Щ$)

$Щ$	$u = 1$; $S > 0$	$u = 2$; $S > 0$	$u = 3$; $S > 0$	$u = 4$; $S > 0$	$u = 5$; $S > 0$	$u = 6$; $S > 0$
$\mathscr{F}_{\text{V}}$ [–1]	1 * 40 + 0 * 72	2 * 40 + 1 * 72	3 * 40 + 3 * 72	4 * 40 + 6 * 72	5 * 40 + 10 * 72	6 * 40 + 15 * 72
	40	152	336	592	920	1320
$\mathscr{F}_{\text{V}}$ [5]	1 * 44 + 0 * 72	2 * 44 + 1 * 72	3 * 44 + 3 * 72	4 * 44 + 6 * 72	5 * 44 + 10 * 72	6 * 44 + 15 * 72
	44	160	348	608	940	1344
$\mathscr{F}_{\text{V}}$ [11]	1 * 56 + 0 * 72	2 * 56 + 1 * 72	3 * 56 + 3 * 72	4 * 56 + 6 * 72	5 * 56 + 10 * 72	6 * 56 + 15 * 72
	56	184	384	656	1000	1416
$\mathscr{F}_{\text{V}}$ [17]	1 * 68 + 0 * 72	2 * 68 + 1 * 72	3 * 68 + 3 * 72	4 * 68 + 6 * 72	5 * 68 + 10 * 72	6 * 68 + 15 * 72
	68	208	420	704	1060	1488
$\mathscr{F}_{\text{V}}$ [23]	1 * 8 + 0 * 72	2 * 8 + 1 * 72	3 * 8 + 3 * 72	4 * 8 + 6 * 72	5 * 8 + 10 * 72	6 * 8 + 15 * 72
	8	88	240	464	760	1128
$\mathscr{F}_{\text{V}}$ [29]	1 * 20 + 0 * 72	2 * 20 + 1 * 72	3 * 20 + 3 * 72	4 * 20 + 6 * 72	5 * 20 + 10 * 72	6 * 20 + 15 * 72
	20	112	276	512	820	1200

$Щ$	$u = 1$; $S < 0$	$u = 2$; $S < 0$	$u = 3$; $S < 0$	$u = 4$; $S < 0$	$u = 5$; $S < 0$	u = 6; $S < 0$
$\mathscr{F}_{\text{V}}$ [–1]	1 * 32 + 0 * 72	2 * 32 + 1 * 72	3 * 32 + 3 * 72	4 * 32 + 6 * 72	5 * 32 + 10 * 72	6 * 32 + 15 * 72
	32	136	312	560	880	1272
$\mathscr{F}_{\text{V}}$ [5]	1 * 28 + 0 * 72	2 * 28 + 1 * 72	3 * 28 + 3 * 72	4 * 28 + 6 * 72	5 * 28 + 10 * 72	6 * 28 + 15 * 72
	28	128	300	544	860	1248
$\mathscr{F}_{\text{V}}$ [11]	1 * 16 + 0 * 72	2 * 16 + 1 * 72	3 * 16 + 3 * 72	4 * 16 + 6 * 72	5 * 16 + 10 * 72	6 * 16 + 15 * 72
	16	104	264	496	800	1176
$\mathscr{F}_{\text{V}}$ [17]	1 * 4 + 0 * 72	2 * 4 + 1 * 72	3 * 4 + 3 * 72	4 * 4 + 6 * 72	5 * 4 + 10 * 72	6 * 4 + 15 * 72
	4	80	228	448	740	1104
$\mathscr{F}_{\text{V}}$ [23]	1 * 64 + 0 * 72	2 * 64 + 1 * 72	3 * 64 + 3 * 72	4 * 64 + 6 * 72	5 * 64 + 10 * 72	6 * 64 + 15 * 72
	64	200	408	688	1040	1464
$\mathscr{F}_{\text{V}}$ [29]	1 * 52 + 0 * 72	2 * 52 + 1 * 72	3 * 52 + 3 * 72	4 * 52 + 6 * 72	5 * 52 + 10 * 72	6 * 52 + 15 * 72
	52	176	372	640	980	1392

$Щ$	$u = 1$; $S > 0$	$u = 2$; $S > 0$	$u = 3$; $S > 0$	$u = 4$; $S > 0$	$u = 5$; $S > 0$	$u = 6$; $S > 0$
$\mathscr{F}_{\text{VII}}$ [1]	1 * 40 + 0 * 72	2 * 40 + 1 * 72	3 * 40 + 3 * 72	4 * 40 + 6 * 72	5 * 40 + 10 * 72	6 * 40 + 15 * 72
	40	152	336	592	920	1320
$\mathscr{F}_{\text{VII}}$ [7]	1 * 52 + 0 * 72	2 * 52 + 1 * 72	3 * 52 + 3 * 72	4 * 52 + 6 * 72	5 * 52 + 10 * 72	6 * 52 + 15 * 72
	52	176	372	640	980	1392
$\mathscr{F}_{\text{VII}}$ [13]	1 * 64 + 0 * 72	2 * 64 + 1 * 72	3 * 64 + 3 * 72	4 * 64 + 6 * 72	5 * 64 + 10 * 72	6 * 64 + 15 * 72
	64	200	408	688	1040	1464
$\mathscr{F}_{\text{VII}}$ [19]	1 * 76 + 0 * 72	2 * 76 + 1 * 72	3 * 76 + 3 * 72	4 * 76 + 6 * 72	5 * 76 + 10 * 72	6 * 76 + 15 * 72
	76	224	444	736	1100	1536
$\mathscr{F}_{\text{VII}}$ [25]	1 * 88 + 0 * 72	2 * 88 + 1 * 72	3 * 88 + 3 * 72	4 * 88 + 6 * 72	5 * 88 + 10 * 72	6 * 88 + 15 * 72
	88	248	480	784	1160	1608
$\mathscr{F}_{\text{VII}}$ [31]	1 * 100 + 0 * 72	2 * 100 + 1 * 72	3 * 100 + 3 * 72	4 * 100 + 6 * 72	5 * 100 + 10 * 72	6 * 100 + 15 * 72
	100	272	516	832	1220	1680

$Щ$	$u = 1$; $S < 0$	$u = 2$; $S < 0$	$u = 3$; $S < 0$	$u = 4$; $S < 0$	$u = 5$; $S < 0$	$u = 6$; $S < 0$
$\mathscr{F}_{\text{VII}}$ [1]	1 * 32 + 0 * 72	2 * 32 + 1 * 72	3 * 32 + 3 * 72	4 * 32 + 6 * 72	5 * 32 + 10 * 72	6 * 32 + 15 * 72
	32	136	312	560	880	1272
$\mathscr{F}_{\text{VII}}$ [7]	1 * 20 + 0 * 72	2 * 20 + 1 * 72	3 * 20 + 3 * 72	4 * 20 + 6 * 72	5 * 20 + 10 * 72	6 * 20 + 15 * 72
	20	112	276	512	820	1200
$\mathscr{F}_{\text{VII}}$ [13]	1 * 8 + 0 * 72	2 * 8 + 1 * 72	3 * 8 + 3 * 72	4 * 8 + 6 * 72	5 * 8 + 10 * 72	6 * 8 + 15 * 72
	8	88	240	464	760	1128
$\mathscr{F}_{\text{VII}}$ [19]	1 * (–4) + 0 * 72	2 * (–4) + 1 * 72	3 * (–4) + 3 * 72	4 * (–4) + 6 * 72	5 * (–4) + 10 * 72	6 * (–4) + 15 * 72
	–4	64	204	416	700	1056

$\mathcal{F}_{VII}$ [25]	1 * (–16) + 0 * 72	2 * (–16) + 1 * 72	3 * (–16) + 3 * 72	4 * (–16) + 6 * 72	5 * (–16) + 10 * 72	6 * (–16) + 15 * 72
	–16	40	168	368	640	984
$\mathcal{F}_{VII}$ [31]	1 * (–28) + 0 * 72	2 * (–28) + 1 * 72	3 * (–28) + 3 * 72	4 * (–28) + 6 * 72	5 * (–28) + 10 * 72	6 * (–28) + 15 * 72
	–28	16	132	320	580	912

4.4. Calculation of Ц

Indice u could be also defined as the iteration rank (see 4.2.1).

Here comes:

$$Ц = i + j.$$

Example for F_V [11], $S < 0$

$$Ц = (-2) + 6u.$$

Rank = u	0	1	2	3	4	Etc...
$i + j$	0	4	10	16	22	Etc...

4.5. Prime condition for P

If we refer to Δ calculation process (§ 4.2.1.), it comes:

$$\Delta^2 = Щ + M' = Ц^2 + M''_P.$$

Which give the prime condition for P:

• P is a composite number if M'_P is egal to a difference between two square numbers (*which, of course, must not be consecutive odd numbers*):

$$M''_P = \Delta^2 - Ц^2,$$

that could be looked as a premise of the Fermat's Last Theorem.

• P is a prime number if M'_P is not egal to a difference between two square numbers.

Regarding above example:

for $P = 2783 \in \mathcal{F}_V$ **[11]** with $N_{2783} = 464$; $M'_{2783} = 312$, $u = 3$, so $\psi_{2783} = (-8)$, it comes:

$$M''_{2783} = M'_{2783} - (-8) = 320$$

$$\Delta = 264 + 312 = 16^2 + 320 = 24^2:$$

Generalization:

1° Due to the remainder, R, in the division of P with 36 as divisor, we know F, the P sub-family

$$P = 36 * k + R.$$

For F_V, $R = \{(-1)$ ou $35, 5, 11, 17, 23, 29, 35\}$.
For F_{VII}, $R = \{1, 7, 13, 19, 25, 31\}$.

It comes M'_P, then ψ, and so M''_P, $Щ$ et $Ц$.

Example:

for F_V [11], $S < 0$, the formula giving M'_P, is:

$$M'_P = (-4)(u - 1) + 4(6u - 1)C_{(1,i)} + 8C_{(2,i)},$$

and

$$M''_P = 4(6u - 1)C_{(1,i)} + 8C_{(2,i)},$$

which is egal to the difference between two square numbers, and, so to the sum of consecutive odd numbers. As $Ц'$ is the last element of this sum, we could check – or not –.

$$(2Ц + 1) + (2Ц + 3) + (2Ц + 5) + ... + (2Ц' - 1),$$

$$M''_P = \Delta^2 - Ц^2.$$

→ If yes, P is a composite number;

→ if not, P is a prime number.

5. EXAMPLES

5.1.

$P = 4439$,

$P = 11 + k * 36$, it comes $P \in \mathcal{F}_V$ [11]; $k = 123$.

$N_{4439} = 740$;

$M'_{4439} = [(2 * (N_{4439} - 2)/3] + 4 = 496$.

M'_{4439} get the general form: $(-16) + 116\boldsymbol{C}_{(1,i)} + 8\boldsymbol{C}_{(2,i)}$.

It comes $\boldsymbol{i}_{4439} = 4$ and $\boldsymbol{u}_{4439} = 5$.

So:

$$\psi = (-16),$$

$$M''_{4439} = M'_{4439} + \psi = 496 + 16 = 512,$$

$$Щ = 5 * 16 + 10 * 72 = 800$$

and

$$Ц = 28,$$

$$M''_{4439} = 57 + 59 + 61 + 63 + 65 + 67 + 69 + 71 = 512 = 36^2 - 28^2.$$

P_{4439} is composite number.

$$Ц' = 36,$$

$$\Delta = Щ + M'_{4439} = Ц^2 + M''_{4439} = 800 + 496 = 28^2 + 512 = 1296 = 36^2,$$

$$i = 4 \text{ and } j = (-32),$$

$$P = 23 * 193.$$

5.2.

$P = 2363$;

$P = 23 + k * 36$, it comes $P \in \mathcal{F}_V$ **[23]** et $k = 65$.

$N_{2363} = 394$;

$M'_{2363} = [(2 * (N_{2363} - 4)/3] + 8 = 268$.

M'_{2363} get the general form: $(-8) + 84\boldsymbol{C}_{(1,i)} + 8\boldsymbol{C}_{(2,i)}$.

It comes $\boldsymbol{i}_{2363} = 3$ and $\boldsymbol{u}_{2363} = 3$.

So:

$$\psi = (-8),$$

$$M''_{2363} = M'_{2363} + \psi = 268 + 8 = 276,$$
$$Щ = 3 * 64 + 3 * 72 = 408$$

and

$$Ц = 20,$$
$$M''_{2363} = 41 + 43 + 45 + 47 + 49 + 51 = 276 = 26^2 - 20^2.$$

P_{2363} is a composite number.

$$Ц' = 26,$$
$$\Delta = Щ + M'_{2363} = Ц^2 + M''_{2363} = 408 + 268 = 20^2 + 276 = 676 = 26^2,$$
$$i = 3 \text{ and } j = (-23),$$
$$P = 17 * 139.$$

5.3.

$P = 1969$;

$P = 25 + k * 36$, it comes $P \in \mathcal{F}_{VII}$ [25] et $k = 54$.

$N_{1969} = 328$;

$M'_{1969} = 16 - (2 * (N_{2363} - 4)/3] = (-200)$.

M'_{1969} get the general form: $40 - 124C_{(1,i)} + 8C_{(2,i)}$.

It comes $|i_{1969}| = 2$ and $u_{1969} = 5$.

So:

$$\psi = 40,$$
$$M''_{1969} = M'_{1969} + \psi = (-200) + (-40) = (-240),$$
$$Щ = 6 * (-16) + 15 * 72 = 984$$

and

$$Ц = 28,$$
$$M''_{1969} = 41 + 43 + 45 + 47 + 49 + 51 = 276 = 32^2 - 28^2.$$

P is a composite number.

$$Ц' = 32,$$
$$\Delta = Щ + M'_{1969} = Ц^2 + M''_{1969} = 984 + (-200) = 32^2 - 240 = 676 = 28^2,$$
$$i = (-2) \text{ et } j = (-30),$$
$$P = 11 * 179.$$

5.4.

$P = 1493$;

$P = 17 + k * 36$, it comes $P \in \mathcal{F}_{V}$ [17] et $k = 41$.

$N_{1493} = 249$;

$M'_{1493} = 2(N - 3)/3) + 9 = 173$.

If P would be a composite number, M'_{1493} must get one of the below general form:

For $i > 0, j < 0$ et $j > i$, which means $S < 0$, it would come for M'_{1493}:

$5 + 16C(1,i) + 8C(2,i)$	$1 + 40C(1,i) + 8C(2,i)$	$(-3) + 64C(1,i) + 8C(2,i)$	$(-7) + 88C(1,i) + 8C(2,i)$	$(-11) + 112C(1,i) + 8C(2,i)$	Etc...

aso...

Or, for $i < 0, j > 0$ et $j > i$, which means $\boldsymbol{S > 0}$, it would come for M'_P:

$9 + 16C(1,i) + 8C(2,i)$	$13 + 40C(1,i) + 8C(2,i)$	$17 + 64C(1,i) + 8C(2,i)$	$21 + 88C(1,i) + 8C(2,i)$	$25 + 112C(1,i) + 8C(2,i)$	Etc...

aso ...

There are no (i, u) pair which could fit with above statement.

$$P = 1493 \text{ is prime.}$$

Due to § 2 and following

$$(i, j) = (3 \pm \sqrt{173})/2$$

it comes:

$6|(i * j)| + (i + j) = 249 = N_{1493}$,

$X = 6 * i - 1 = 6 * (3 + \sqrt{173})/2 - 1$,

$Y = 6 * j + 1 = 6 * (3 - \sqrt{173})/2 + 1$,

$X * Y = \{[6 * (3 + \sqrt{173})/2] - 1\} * \{[6 * (3 - \sqrt{173})/2] + 1\} = 1493$.

CONCLUSION

Prime rule for integers. An integer is prime when its co-second indice is not egal to the difference of two square numbers, X^2 and Y^2, $\forall X$ et $Y \in \mathbb{N}^*$, X or Y not odd consecutive numbers, and $X, Y \neq 1$. P is prime if: $M''_P \neq Y^2 - X^2$. Also, regarding Stirling's formula:

$$M''_P = C_{(1,Y)} - C_{(1,X)} + 2[C_{(2,Y)} - C_{(2,X)}].$$

REFERENCES

1. *André Maïsseu, Benoît Maïsseu*. Вестник НИЯУ МИФИ. 2019. Т. 8. № 2. С. 175–178.

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Базовые условия для простых целых чисел

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В этой статье мы предлагаем разбить натуральные числа P на два коэффициента i и j, сообщаемые их нижнему индексу N, который определяется уравнением $P = 6\ N \pm 1$; i и j определены другим уравнением, $N = 6 * |i| * |j| \pm (i \pm j)$. N является натуральным целым числом, но i и j не обязательно являются натуральными целыми числами. Они могут быть иррациональными. Используя такой необычный подход теории чисел, мы предлагаем простое соотношение между двумя квадратами чисел в качестве необходимого условия для любого простого числа. Мы предполагаем, что такие отношения можно рассматривать как следствие последней теоремы Ферма.

Ключевые слова: натуральное целое число, простые числа, составное число, рациональное число, иррациональное число, теорема Ферма

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СПИСОК ЛИТЕРАТУРЫ

1. André Maïsseu, Benoît Maïsseu. Вестник НИЯУ МИФИ. 2019. Т. 8. № 2. С. 175–178.